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this time: probability

next time: inference

* HW 2 due by 11:59 pm on Fri 4 May 18

AMS 7
24 Apr 18

The Meaning of Probability

Pascal / Fermot (1660)

- frequentist approach (relative frequency): attention is restricted to phenomena that are repeatable under identical conditions w/ independent trials

  P(A): long-run relative frequency

Thomas Bayes ($\sim 1700 \rightarrow \sim 1760$)

- Bayesian approach: A can be any (true/false) proposition

  P(A): numerical measure of the weight of evidence in favor of the statement that A is true

$\boxed{\text{Equally Likely Model (ELM)}}$

if all outcomes are equally likely,

$$P(A) = \frac{\text{number of outcomes favorable to A}}{\text{total number of possible outcomes}}$$

population $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}}$ sample $\begin{bmatrix} Y_1 \end{bmatrix}$ $n=1$

$\boxed{\text{ELM?}}$ Yes, because sampling was at random

$P(Y_1 \text{ is odd}) = ?$

PF $P(Y_1 \text{ is odd}) = \frac{2}{3}$

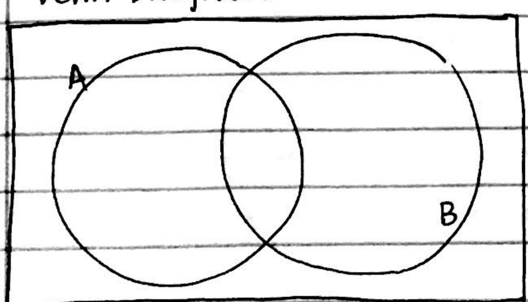
$\boxed{\text{ELM?}}$ Yes, so P(any one of their child is T-S) $= \frac{1}{4} = 25\%$

P(A), P(not A) ?

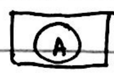
P(A or B) ?

P(A and B) ?

Venn Diagram



$P(A) = \frac{A}{\boxed{1}}$

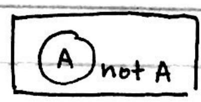


* easy rule: ① for any T/F statement A,

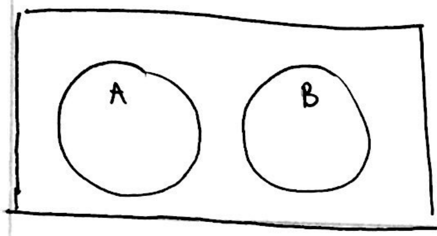
$$0\% = 0 \le P(A) \le 1 = 100\%$$

② $P(A) + P(\text{not } A) = 1 = 100\%$

$P(A) = 1 - P(\text{not } A)$



$\boxed{P(A \text{ or } B)}$

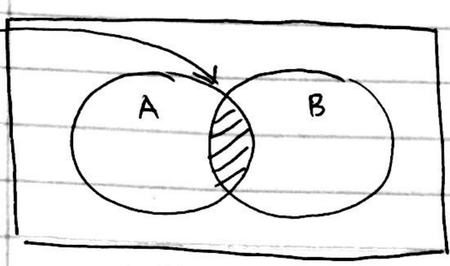


for this diagram:

$P(A \text{ or } B) = P(A) + P(B)$

addition rule for or with mutually exclusive A, B

overlap (A & B)



for this diagram:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

general addition rule for or

$\boxed{P(A \text{ and } B)}$

population $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}}$ sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n=2$

$P(Y_1 = 2 \text{ and } Y_2 = 2) = ?$

population $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{with replacement}]{\text{at random}}$ sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n=2$

IID sampling
↳ independent
identically
distributed

$P(Y_1 = 2 \text{ and } Y_2 = 2) = ?$

		Y_2: 1	2	9
	1	(1,1)	(1,2)	(1,9)
Y_1	2	(2,1)	(2,2)	(2,9)
	9	(9,1)	(9,2)	(9,9)

$\boxed{\text{ELM?}}$ Yes

$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$P(Y_1 = 2) = \frac{3}{9} = \frac{1}{3}$

$P(Y_2 = 2) = \frac{3}{9} = \frac{1}{3}$

P(1 or more T-s)

# T-s babies	if ~~ELM~~
0	
1	
2	
3	
4	
5	

not equally likely

P(1 or more) $= \frac{5}{6}$

conjecture:

$P(A \text{ and } B) = P(A) \cdot P(B)$

pop $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{without replacement}]{\text{at random}}$ sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ simple random sampling (SRS)

$P(Y_1 = 2 \text{ and } Y_2 = 2) = 0 \leftarrow$ common sense

$P(Y_1 = 2) = \frac{1}{3}$; $P(Y_2 = 2) = \frac{1}{3}$

SRS

		Y_2: 1	2	9
	1	$\times$	1,2	
Y_1	2		~~2,2~~	
	9		9,2	$\times$

$\boxed{\text{ELM?}}$ Yes

$P(Y_2 = 2) = \frac{2}{6} = \frac{1}{3}$

here $P(Y_1 = 2 \text{ and } Y_2 = 2) = 0$

but $P(Y_1 = 2) \cdot P(Y_2 = 2) = \frac{1}{9} \neq 0$

next time: Conditional Probability