read:
DD ch. 1–11 (B)
1–3 (A)
LN pp. 127–156
today:
L–119

this time: probability models for sums
next time: means; inference

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AMS 7
1 May 18

Thurs: midterm handed out on 3 May in class
due Sun 13 May 11:59 pm
I will hold extra office hours Fri, Sat, Sun, Mon ... in Jack's Lounge

Roulette
R–52

· people try to maximize
utility function = (money, satisfaction)

$$\sigma = \sqrt{\frac{[(-1)-(-0.05)]^2 \cdot 37 + [1(+35)-(-0.05)]^2}{38}}$$

math fact: if population contains only 2 values,

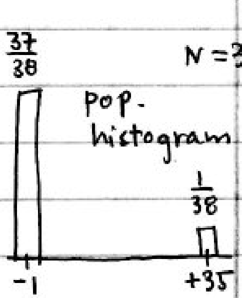
$$\sigma = \left[\begin{pmatrix}\text{larger}\\ \text{value}\end{pmatrix} - \begin{pmatrix}\text{smaller}\\ \text{value}\end{pmatrix}\right]\sqrt{\begin{pmatrix}\text{proportion}\\ \text{of}\\ \text{larger values}\end{pmatrix}\begin{pmatrix}\text{proportion}\\ \text{of}\\ \text{smaller values}\end{pmatrix}}$$

here,

$$\sigma = \underbrace{[(+35)-(-1)]}_{36}\sqrt{\frac{1}{38}\cdot\frac{37}{28}} = \$5.76$$

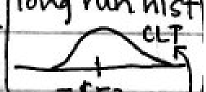
* On average, on each \$1 bet on a single #, I expect to (win ≈ −0.05) (lose = \$0.05), give or take about σ = \$5.76.

pop	Ⓐ: single # $\boxed{6}$	sample		imaginary dataset
possible spins		the observed spins		all possible values of S'
your net gain		your net gain		your net gain
N = 38: [37; −1; 1; +35]	IID →	[mostly −1, ever so often +35]	n = 1,000 → IID	[−64; −28; ⋮] S' ⋮ M → ∞
mean μ = −0.05		sum S' = ?		sum S' = ?
SD σ = \$5.76		ex. −64		ex. −28



long run mean
expected value of S' = −\$53

long run SD
standard error of S' = \$182

long run hist


L–124

[Your net gain after 1,000 \$1 bets on a single #] (the world) is like [the sum S' of n = 1,000 IID draws from pop p*] (our probability model of the world)

P(coming out ahead) = P(S' > 0) = ?

$\frac{1}{38} \cdot 1000 \doteq 26$ wins → \$910
974 losses → −974
1000 −64

27 wins → +945
973 losses → −973
−28

$\left(\begin{array}{c}\text{expected}\\ \text{value}\\ \text{of } \$'\end{array}\right) = \text{EV of } \$' = \boxed{E_{IID}(\$') = n\mu}$

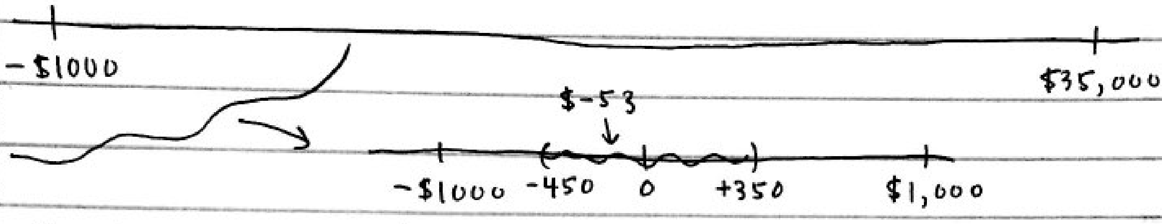
$= (\#\text{ draws}) \cdot (\text{pop mean})$

possible ingredients	belong?
N	no
μ	$\mu \uparrow \ \$ \uparrow$
σ	no
n	yes

here,

$E_{IID}(\$') = (1000)(-0.0526)$

$= \$-52.60$



* after $n = 1{,}000$ \$1 plays on a single #, you expect to have won (lost about \$53) $E_{IID} = -53$, give or take

$\left(\begin{array}{c}\text{standard}\\ \text{error of } \$'\end{array}\right) = \text{SE of } \$' = \boxed{SE_{IID}(\$') = \sigma\sqrt{n}}$

$= (\text{pop SD})\sqrt{\#\text{ draws}}$

possible ingredients	belong?
N	no
μ	no
σ	$\sigma \uparrow$ SE$\uparrow$
n	n$\uparrow$ SE$\uparrow$

σ = "noise level" in each draw from pop.

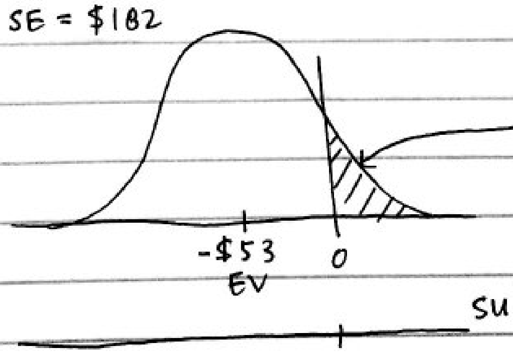
$SE(\$) = (\text{pop SD})\sqrt{\#\text{ draws}} = (\$5.76)\sqrt{1000}$

$\doteq \$182$

long run histogram of \$' [A]

SE = \$182

$z = \dfrac{0-(-53)}{182} = +0.29$



P(coming out ahead, strategy [A]) = 39%

long run histogram of \$' [B]

SE = \$127

P(\$ > 0) = 34%