read:
LN pp. L-214 → L-282
today:
LN pp L-214 → L-221

this time: correlation & regression
next time: ANOVA

* HW 3 due by Sun 27 May 18 on canvas

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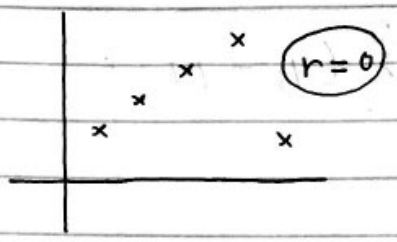
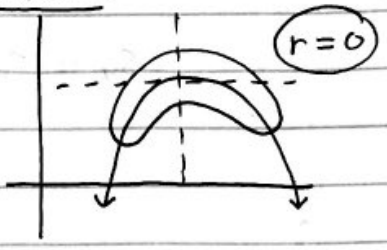
AMS 7
24 May 18

correlation coefficient (r)

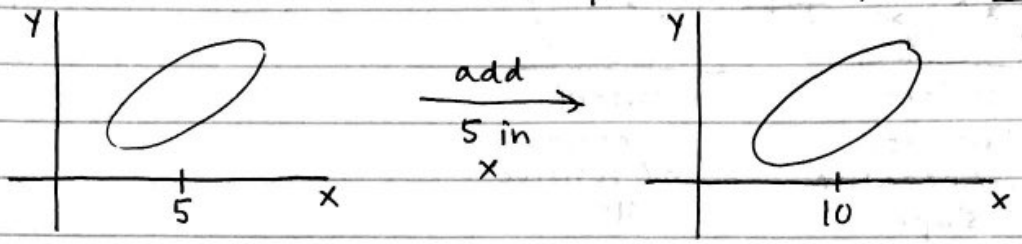
★ r can be fooled by outliers



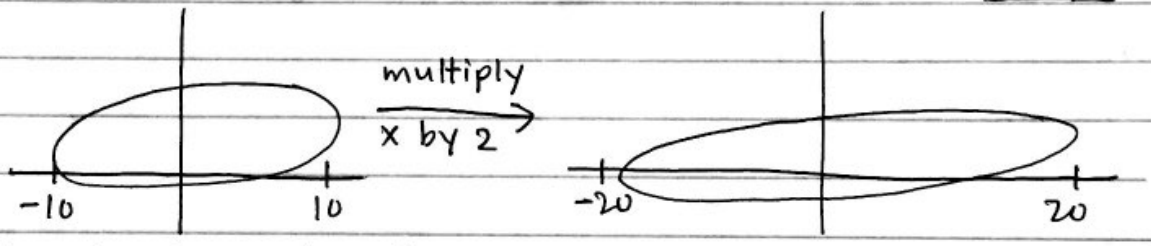


* especially when n is small

★ if a constant is added to y or x values, r is unchanged



★ if a constant is multiplied to y or x values, r is changed.

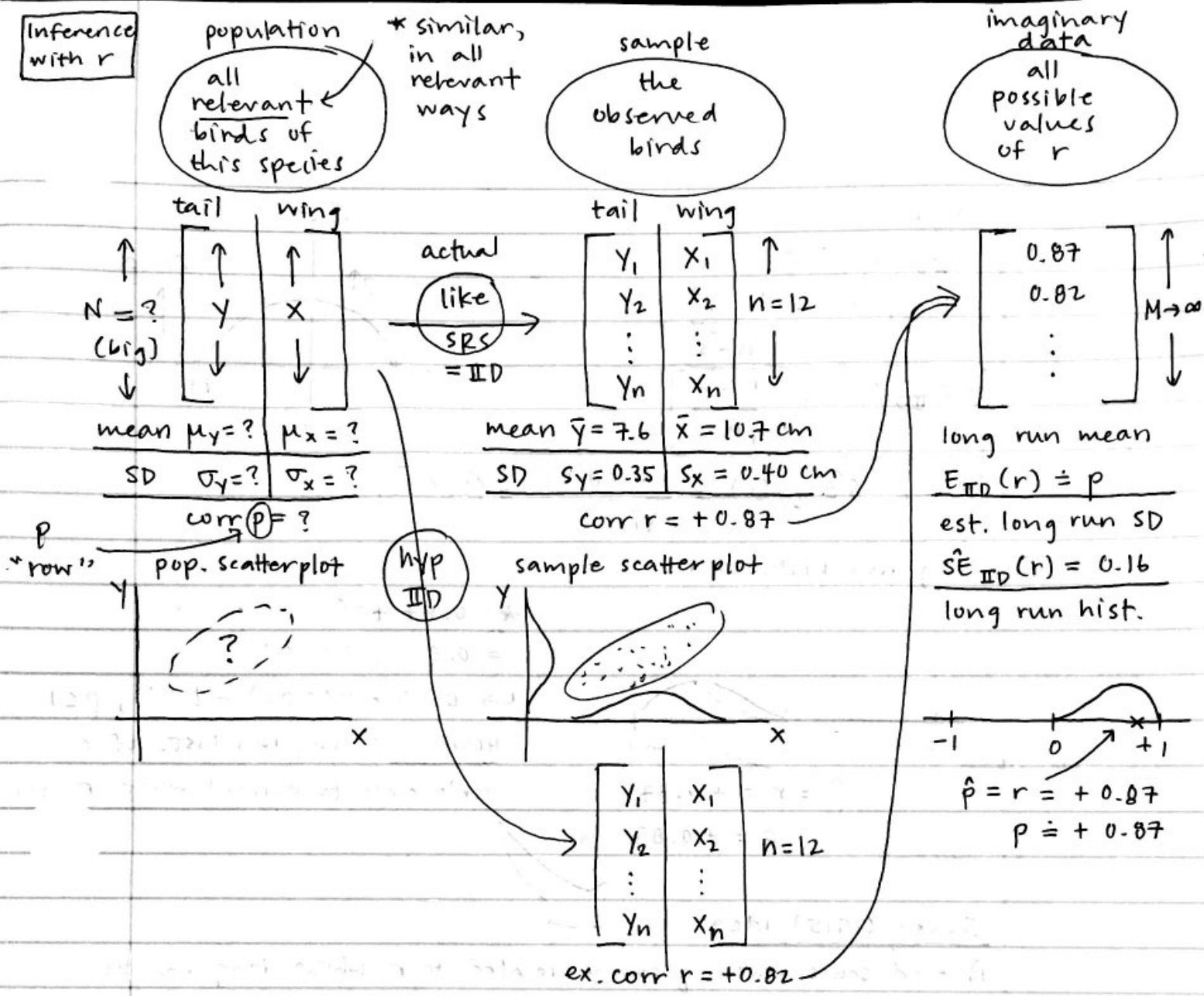


★ $r(x, y) = r(y, x)$

★ $r(x, x) = 1$

Devil's Advocate: the only reason we got $r \neq 0$ is because of unlucky random sampling
– must take seriously

Inference with r



Inferential Summary

pop	unknown pop. quantity of main interest	ρ = pop. correlation between wing & tail length of this species
sample	estimate of ρ	r = +0.87
↑ imag data ↓	give or take for r as est. of ρ	$\hat{SE}(r)$ = 0.16
	95% CI for ρ	approx: (0.55, 1.0) exact: (0.59, 0.96)

facts:

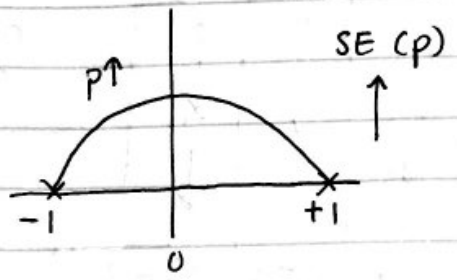
① $E_{\text{IID}}(r) \doteq \rho$

② $SE_{\text{IID}}(r) = \dfrac{1-\rho^2}{\sqrt{n-3}}$

$\hat{SE}_{\text{IID}}(r) = \dfrac{1-r^2}{\sqrt{n-3}}$

here, $\hat{SE}_{\text{IID}}(r) = \dfrac{1-(0.8)^2}{\sqrt{12-3}} = 0.081$

$n\uparrow \; SE(r)\downarrow$



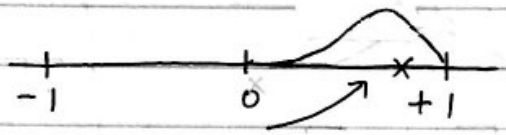
③ long run hist. of r

★ $r \pm 2\hat{SE}(r)$

$= 0.87 \pm 2(0.08)$

↳ $0.87 + 2(0.08) \doteq 1.03$, $\rho \le 1$

HOWEVER: long run hist. of r isn't close to normal when $r = \pm 1$



$\hat{\rho} = r = +0.87$

$\rho = +0.87$

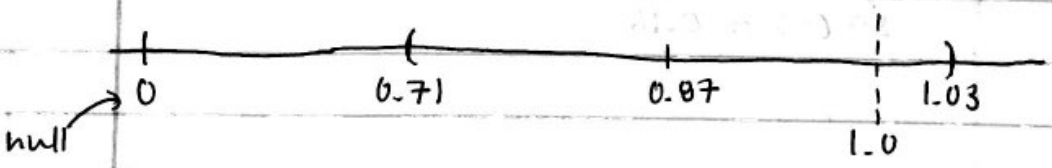
Fisher (1915) Idea

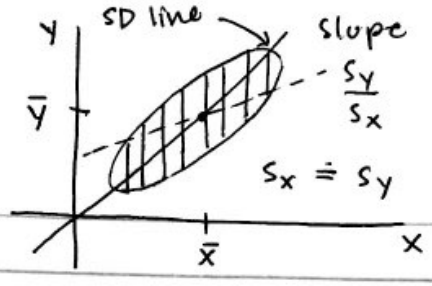
1) find some other quantity related to r whose long run hist. does follow the normal curve
2) do est. ± 2SE story on the derived quantity
3) back-transform to find r

★ $0.87 \pm 2(0.08) = (0.71, 1.0)$

approx 95% CI for ρ

STATSIG ✓

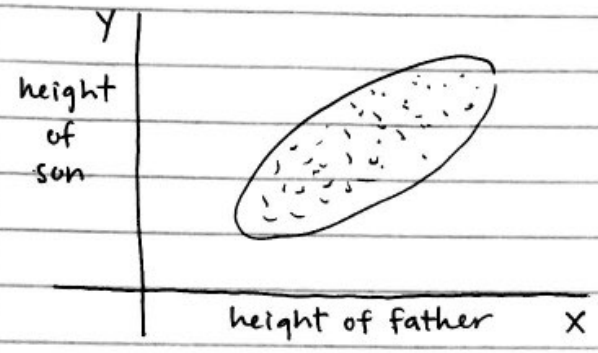




$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

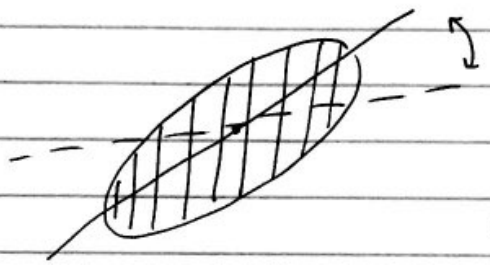
$\hat{y}$: predicted y-value; $\hat{\beta}_0$: est. intercept; $\hat{\beta}_1$: est. slope

Galton 1890



n = 1,000 UK families with at least one son

★ regression to mediocrity



- tall fathers tend to have sons not as tall as them
- short fathers tend to have sons not as short as them

* the extremes fall back to place

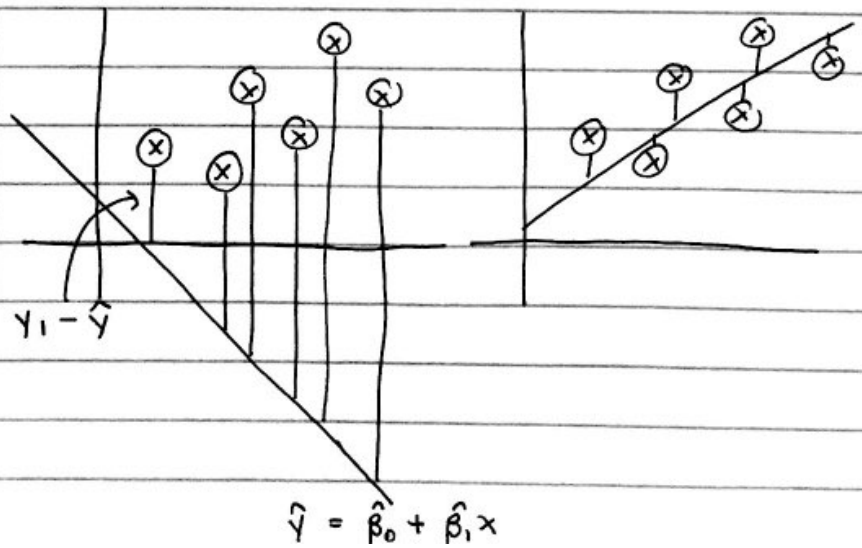
$$\hat{\beta}_1 = r \frac{s_x}{s_y}$$

$$\hat{\beta}_1 = (0.8704)\frac{0.3499 \text{ ← cm of TL}}{0.3950 \text{ ← cm of WL}} = 0.771 \frac{\text{cm of TL}}{\text{cm of WL}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 7.567 \text{ cm of TL} - \left(0.771 \frac{\text{cm of TL}}{\text{cm of WL}}\right)(10.68 \text{ cm of WL})$$

$$= \boxed{-0.669 \text{ cm of TL}}$$

Gauss 1800



find $(\hat{\beta}_0, \hat{\beta}_1)$ to minimize

$$\sum_{i=1}^{n} (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x))^2$$

↳ regression line = $(\hat{\beta}_0, \hat{\beta}_1)$ (least squares line)