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AMS 7
3 May 18

this time: probability models for means

next time: inference

★ Get more good data
↳ unbiased

Measurement Error

$$\begin{bmatrix} 16 \\ 16 \\ 16 \\ \vdots \\ 16 \end{bmatrix} \quad \begin{bmatrix} 16.0 \\ 16.0 \\ 16.0 \\ \vdots \\ 16.0 \end{bmatrix} \quad \Bigg| \quad \begin{bmatrix} 16.03 \\ 15.99 \\ 15.95 \\ \vdots \\ \ \end{bmatrix}$$

non-random (deterministic) — probablistic (random) (stochastic)

astronomy (1600s)

Basic Measurement Error Model

$$\left.\begin{aligned} \begin{pmatrix}\text{obs.}\\ \#1\end{pmatrix} &= \begin{pmatrix}\text{true}\\ \text{value}\end{pmatrix} + (\text{bias}) + \begin{pmatrix}\text{random}\\ \text{error }\#1\end{pmatrix} \\ \begin{pmatrix}\text{obs.}\\ \#2\end{pmatrix} &= \begin{pmatrix}\text{true}\\ \text{value}\end{pmatrix} + (\text{bias}) + \begin{pmatrix}\text{random}\\ \text{error }\#2\end{pmatrix} \\ &\ \ \vdots \\ \begin{pmatrix}\text{obs.}\\ \#n\end{pmatrix} &= \begin{pmatrix}\text{true}\\ \text{value}\end{pmatrix} + (\text{bias}) + \begin{pmatrix}\text{random}\\ \text{error }\#n\end{pmatrix} \end{aligned}\right\}$$

IID draws from a normal curve with mean = 0 & SD σ

$$y_1 = \theta + b + e_1$$
$$y_2 = \theta + b + e_2$$
$$\vdots$$
$$y_n = \theta + b + e_n$$

$$16.03 = 16.0 + 0 + (+0.03)$$
$$(y_1) = \theta + \underset{b}{(\text{unbiased})} + e_1$$

$$15.99 = 16.0 + 0 + (-0.01)$$
$$(y_2) = \theta + b + e_2$$

$$\vdots$$

$$15.95 = 16.0 + 0 + (-0.05)$$
$$(y_n) = \theta + b + e_n$$

$$
\begin{aligned}
Y_1 &= \theta + b + e_1 \quad \leftarrow \text{IID with mean 0} \\
Y_2 &= \theta + b + e_2 \\
&\vdots \\
Y_n &= \theta + b + e_n \\
\hline
\bar{Y} &= \theta + b + \bar{e}
\end{aligned}
$$

* $\bar{e}$ will (with high prob.) be closer to 0 than any of the e_i $\left(\text{mean of } n \text{ IID draws, each with mean } 0\right)$ ex. $\dfrac{(+0.03)+(-0.01)+ \ldots + (-0.05)}{n}$

* as $n \uparrow$, $\bar{e} \downarrow 0$ with high prob.; therefore, $\bar{Y}$ will be close (when n is large) to $(\theta + b)$ (truth + bias)

* to make $\bar{Y}$ get arbitrarily close to θ, we need 2 things:
  1. n should get $\uparrow$
  2. $b = 0$ (measuring process is <u>unbiased</u>)

* you cannot make bias $\downarrow 0$ as n increases

1936
Roosevelt (D)
Landon (R)

Literary Digest
- 24,000 letters (pre-stamped postcards)
- got back 16,000 letters
- LD est. that (Landon 60%, Roosevelt 40%)

truth:

(Roosevelt 60%, Landon 40%) $\rightarrow$ 20 - percentage point error

George Gallup

(Iowa State) (1,000 people)

addresses:
- landowner records
- telephone books
- club membership

} rich

population (conceptual) all possible measurements

sample: the observed measurements

imaginary dataset

potassium level

$N = \infty$

mean $\mu = 3.8$

SD $\sigma = 0.2$

like $\xrightarrow{\text{IID}}$

hyp IID

potassium level

y_1, y_2, $\vdots$, y_4

actual $n = 4$

mean $\bar{y} = ?$

ex. 3.94

$n = 4$

mean $\bar{y} = ?$

ex. = 3.77

3.92 ($\bar{y}$)

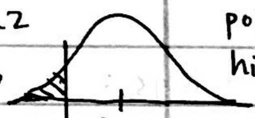
3.77 ($\bar{y}$)

$\vdots$

$M \to \infty$

long run mean	expected value of $\bar{y}$
long run SD	standard error of $\bar{y}$
long run hist.	3.8 EV

SD = 0.2



pop-hist. = hist of one reading at a time (y_1)

$$\frac{3.5 - 3.8}{0.2} = \frac{-0.3}{0.2} = -1.5$$

7% $\boxed{\text{L-133}}$

P(misdiagnosis with $n = 1$) $\doteq$ 7% ← error rate too high

P(misdiagnosis with $n = 4$) $= P(\bar{y}_{(n=4)} < 3.5) = ?$

$$\begin{pmatrix}\text{expected}\\ \text{value}\\ \text{of } \bar{y}\end{pmatrix} = (\text{EV of } \bar{y}) = \boxed{E_{IID}(\bar{y}) = \mu}$$

$$\begin{pmatrix}\text{standard}\\ \text{error of}\\ \bar{y}\end{pmatrix} = (\text{SE of } \bar{y}) = \boxed{SE_{IID}(\bar{y}) = \frac{\sigma}{\sqrt{n}}}$$

ingredient	belong?
N	X
μ	X
σ	$\sigma\uparrow$ $SE(\bar{y})\uparrow$
n	$n\uparrow$ $SE(\bar{y})\downarrow$

Square Root Law:

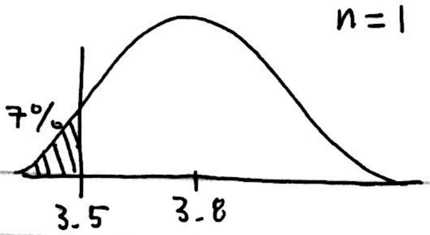
- uncertainty goes down with n, but only at a $\sqrt{n}$ rate
- to cut $SE(\bar{y})$ in half, need to quadruple n

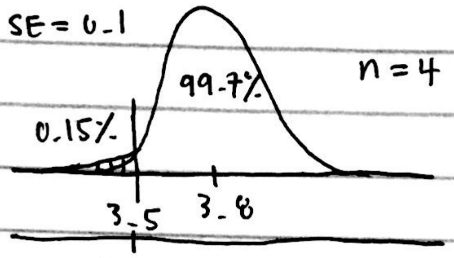
$$SE(\bar{y}) = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{4}} = 0.1$$

Scanned with CamScanner

L-124

SE = 0.2 CLT





$$\frac{3.5 - 3.8}{0.1} = -\frac{0.3}{0.1}$$

n	P(misdiagnosis)	cost
1	7%	\$25
4	0.15%	\$100

cost - benefit tradeoff