N.N

AMS 7 · Lecture 5·1·18 (1)

THIS TIME: Prob. models for sums
NEXT TIME: means ; inference

* Read : DD ch. 1−11 (B)
1−3 (A)
L.N. Pg 127 → 156

Today
L−119 → ...

* Midterm will be handed out on May 3rd in class (due May 13th @11:59pm)
* extra office hours will be held Fri, Sat, Sun, mon, ... in Jack's Lounge

(R−52) ▷ **Roulette**

losses = negative gains

Population ⇒ Greek letters — Sample ⇒ ordinary alphabet

↳ μ = mean & SD = σ — ↳ $\bar{y}$ = mean

* People try to maximize utility (money, satisfaction)
↳ average utility function

* on average we expect to lose a nickle ($\mu = -0.05$)

$$\sigma = \sqrt{\frac{[(-1)-(-0.05)]^2 \cdot 37 + 1[(+35)-(-0.05)]^2}{38}}$$

Math FACT : If population contains only 2 values,

$$\sigma = \left[\begin{pmatrix}\text{larger}\\\text{value}\end{pmatrix} - \begin{pmatrix}\text{smaller}\\\text{value}\end{pmatrix}\right]\sqrt{\begin{pmatrix}\text{Proportion of}\\\text{larger values}\end{pmatrix}\cdot\begin{pmatrix}\text{Proportion of}\\\text{smaller values}\end{pmatrix}}$$

Hence, ...

$$\sigma = [(+35)-(-1)]\sqrt{\frac{1}{38}\cdot\frac{37}{38}} = \$5.76$$

* on average, on each \$1.00 bet on a single #, I expect to win $\mu = \$-0.05$ (lose a nickle) give or take about $\sigma = \$5.76$

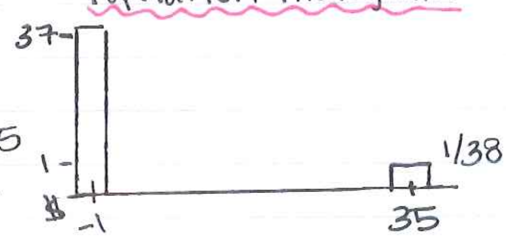
Population: P* — Possible Spins — Your Net Gain (The world)

N=38

$$\begin{bmatrix} 37 & -\$1 \\ 1 & +\$35 \end{bmatrix}$$

mean $\mu = \$-0.05$
SD $\sigma = \$5.76$

Population Histogram



Gamble A : Single #6

Sample : observed spins

Your Net Gain

IID

$n = 1{,}000$

[mostly $-\$1$ and ever so often $+\$35$]

Sum S = ?

↳ our probability model of the world

* your net gain after a 1,000 \$1 bets on a single # is like the sum S of $n = 1{,}000$ IID draws from the Roulette population (P^*)

Possible S = \$ -64

Probability

P(coming out ahead) = $P(S > \$0)$ = ?

$\frac{1}{38} \cdot 1000 \doteq 26$ wins → \$910

$\frac{974}{1000}$ losses -974 / -64

* 27 wins just by luck... $27 \cdot \$35 = \$945$

973 losses -973 / -28

$64 + (-28) = \$36$ (distance b/w $^+35$ & -1)

All possible values of S in Imaginary Data Set

L-124

M ↑↓
$$\left[\begin{array}{c} \$-64 \\ \$-28 \\ \vdots \end{array}\right] \begin{array}{l} = S \\ = \text{another } S \\ \end{array}$$

expected value of S = $\$-53$

standard error of S = \$182

CLT ↙

long run histogram

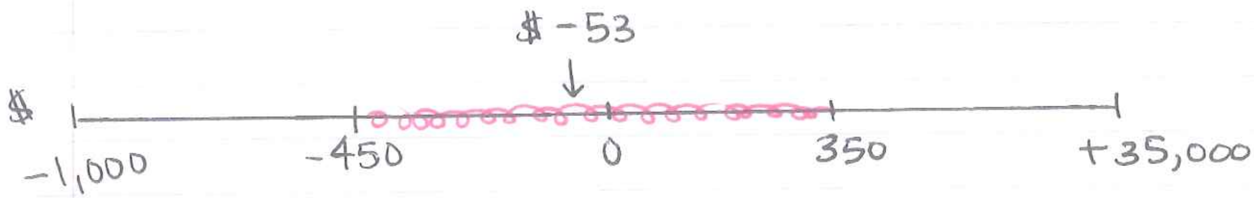
the normal curve

$M \to \infty$ or big # ← long run mean = expected value of S = E.V. of S = $E_{IID}(S) = n\mu$

expected value of S = $\left(\#\text{ of draws}\right) \cdot \left(\text{Pop. mean}\right) = n\,\mu$

Possible Ingredients	belong?	
N	no	x
μ	yes	as μ goes up, S goes up
σ	no	x
n	yes	yes

$E_{IID}(S) = (1000)(-0.0526) = -52.60 \doteq \$-53.0$



→ After n = 1,000 $1 plays on a single #, you expect to have won about $E_{IID}(S) = \$-53$, give or take

$\left(\text{standard error of } S\right) = \underbrace{SE}_{\text{standard error}} \text{ of } S = SE_{IID}(S)$

Possible Ingredients	belong?
N	no
μ	no
σ	$\sigma\uparrow$ SE$\uparrow$
n	n$\uparrow$ SE$\uparrow$

$SE(S) = \left(\text{Pop. SD}\right)\sqrt{\#\text{ of draws}}$

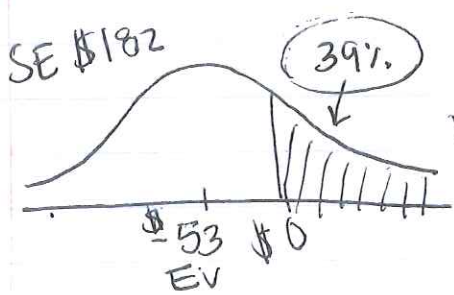
$= \sigma\sqrt{n}$

$= (85.76)\sqrt{1000}$

$= \$182$

σ = "noise level" in each draw from the population

* review L-124 for CLT (central Limit Theorem)



long run histo of S

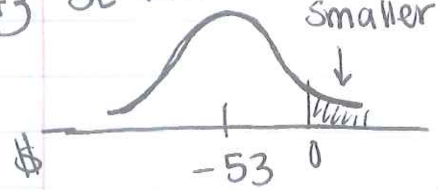
SU

$\dfrac{\$0 - (\$-53)}{\$182} = +0.29$

P(coming out ahead w/ strategy A) = 39%

L-127 SE $127

34%
0.42
Smaller



ditto B
P (<$70) = 34%.
Strategy A better than B