AMS 7 - Lecture 4·24·18

THIS TIME: Probability
NEXT TIME: Inference

HW # 2
due by 11:59 pm
Fri 4 May 18th

▷ ANNOUNCEMENTS

* Midterm will be handed out May 3rd (Thurs)
  Will be due on May 11th
* HW # 3 assigned on May 14th
  Will be due on May 25th
* HW # 4 assigned on May 28th
  Will be due June 8th
* Take Home Final handed out June 7th
  Will be due on June 15th

3.1 ▷ The Meaning of Probability

L-97

→ case study (genetics): Tay-Sachs (T-S) disease

- Two main ways to think about probability
  → the frequentist (or relative frequency) approach
  → the Bayesian approach

L-98

  * Thomas Bayes (~1700 - 1760)
- Pascal / Fermot (1660)

* Equally Likely Model (ELM)

$$\underset{\text{Population}}{\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}} \xrightarrow[\text{random}]{\text{Draw at}} \underset{\text{Sample}}{[\,Y_1\,]} \quad n = 1$$

>> What is the probability that Y_1 is an odd number?
ELM applies here, because sampling was at random
↳ The probability that Y_1 is an odd #
PF) is 2/3 $\left[P(Y_1 \text{ is odd}) = \frac{2}{3} \right]$

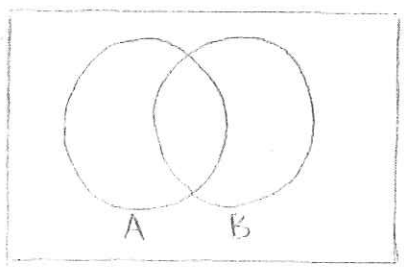
→ In genetic Study
ELM? Yes, so P(anyone of their children is T-S)
is $\frac{1}{4} = 25\%$

- $P(A \text{ or } B) = P(A)\ ?\ P(B)$

* Venn Diagrams

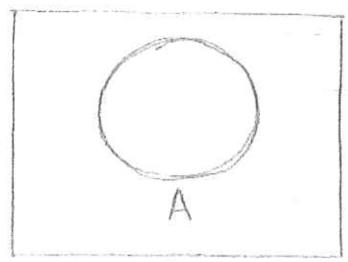
↳ helps with working out (intuitively) how $\boxed{\text{and}}$, $\boxed{\text{or}}$, & $\boxed{\text{not}}$ behave

L-102



Venn Diagram example

Simple Form

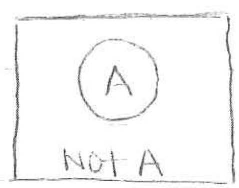


$P(A) = \frac{Ⓐ}{\boxed{1}}$ ← area

* Easy rule: For any T/F statement A,

① $0\% = 0 \le P(A) \le 1 = 100\%$

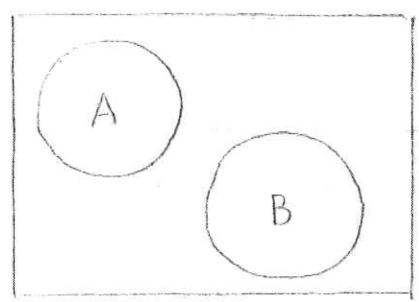
② $P(A) + P(\text{not } A) = 1 = 100\%$



$\boxed{\text{NOT}}$ $P(A) = 1 - P(\text{not } A)$

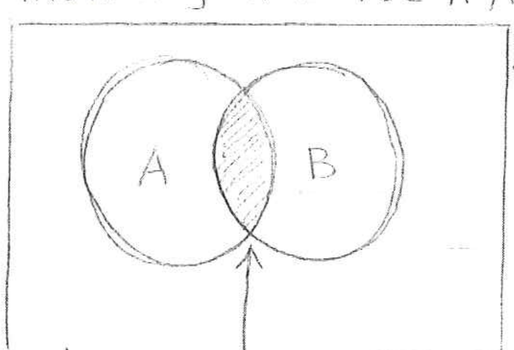
$\boxed{\text{OR}}$ Mutually exclusive = no overlap

For this diagram, $P(A \text{ or } B)$ — $P(A) + P(B)$ →



→ Addition rule for or with mutually exclusive A, B

↳ no overlap ∴ $P(A \& B) = 0$



overlap (A and B)

← General Addition rule for OR

For this diagram,

$P(A \text{ or } B) =$

$P(A) + P(B) - P(A \text{ and } B)$

$\boxed{\text{AND}}$ Pop. of interest $\to$ Sample

$$\begin{bmatrix} 2 \\ 1 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad n = 2$$

$P(Y_1 = 2 \text{ AND } Y_2 = 2) = ?$

* At random with replacement

↳ IID : independent identically distributed

IID table

Y_1 \ Y_2	1	2	9
1	(1, 1)	(1, 2)	(1, 9)
2	(2, 1)	(2, 2)	(2, 9)
9	(9, 1)	(9, 2)	(9, 9)

ELM? yes

$P(Y_1 = 2 \text{ and } Y_2 = 2) = \frac{1}{9}$

$P(Y_1 = 2) = \frac{3}{9} = \frac{1}{3}$

$P(Y_2 = 2) = \text{''} = \text{''}$

P(1 or more T-S)

# of T-S babies

0

1, 2, 3, 4, 5

NOT EQUALLY LIKELY

ELM does not apply here

$P(1 \text{ or more}) = \frac{5}{6}$

Conjecture: $P(A \text{ AND } B) = P(A) \cdot P(B)$

Common sense ↓

Pop. $\to$ Sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random (without replacement)}]{\text{at}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad n = 2$$

$P(Y_1 = 2 \; \& \; Y_2 = 2) = 0$

$P(Y_1 = 2) = \frac{1}{3}$

$P(Y_2 = 2) = \frac{1}{3}$

↳ simple random sampling (SRS)

SRS table

Y_1		1	2	9
	1	~~(1,1)~~	(1,2)	(1,9)
	2	(2,1)	~~(2,2)~~	(2,9)
	9	(9,1)	(9,2)	~~(9,9)~~

ELM? yes

$P(Y_2 = 2) = \frac{2}{6} = \frac{1}{3}$

*because of SRS, (1,1), (2,2), & (9,9) not possible

here $P(Y_1 = 2 \text{ and } Y_2 = 2) = 0$

but $\underbrace{P(Y_1 = 2)}_{\frac{1}{3}} \cdot \underbrace{P(Y_2 = 2)}_{\frac{1}{3}} = \frac{1}{9} \neq 0$

We need a new kind of probability for dependence

↳ Conditional Probability