AMS 7 - Lecture 5.8.18

THIS TIME → inference for \( \mu \)
NEXT TIME → inference for \( \pi \)

Read → LN pg. 137-140
DD ch. 1-11 (B)
DD ch. 1-3 (A)

today → pg. 137 →

4.1 Confidence Intervals
* case study (marine biology)

* Question 1:
Is the difference between \( 25.0^\circ \text{C} \) (\( \bar{Y} \)) & \( 24.3^\circ \text{C} \) (theoretical mean) practically significant?

\[ \Rightarrow \text{large in biological terms} \]

* Answer 1:
( Best ) consult an expert on intertidal crabs
(helpful)

\[ \frac{25.0^\circ \text{C} - 24.3^\circ \text{C}}{24.3^\circ \text{C}} = 2.9\% \]

* Is a 2.9\% diff. between data & theory big enough to matter?

\[ \Rightarrow \text{Rough rule of thumb:} \]
Relative differences of 5\% or more are often, but not always, practically; be practically if they accumulate over time.

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**Intertidal Crabs**

\[ \begin{align*}
\text{Population} & \quad (\text{All crabs similar to those in sample in all relevant ways}) \\
\text{Sample} & \quad (\text{the observed crabs}) \\
\text{Imaginary data} & \\
\end{align*} \]

\[ \begin{align*}
N = ? & \quad \text{(Is the sample size large enough?)} \\
M = ? & \quad \text{(Is the mean of the population known?)} \\
\sigma = ? & \quad \text{(Is the standard deviation known?)} \\
\end{align*} \]

Like SRS \( \Rightarrow \) IID

<table>
<thead>
<tr>
<th>Sample</th>
<th>\text{e. temp}</th>
<th>\begin{bmatrix} 25.8 \ \vdots \ 26.4 \end{bmatrix}</th>
<th>\text{mean } \bar{Y} = 25.0^\circ \text{C}</th>
<th>\text{SD } s = 1.34^\circ \text{C}</th>
</tr>
</thead>
</table>

\[ \begin{align*}
25.8 & \quad h = 26 \\
\end{align*} \]
To specify population, answer: What is the broadest scope of valid generalizability outward from the sample data set?

**Hypokalemia**

Population $\xrightarrow{IID}$ Sample

<table>
<thead>
<tr>
<th>Whole Known</th>
<th>Part Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Specific</td>
</tr>
<tr>
<td>$M=\text{known}$</td>
<td>$\bar{Y}=\text{unknown}$</td>
</tr>
</tbody>
</table>

(deductive reasoning) = (deduction)

**CRABS**

Population $\xrightarrow{IID}$ Sample

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<th>Part Known</th>
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<td>Particular</td>
</tr>
<tr>
<td>$M=\text{unknown}(?)$</td>
<td>$\bar{Y}=\text{known}$</td>
</tr>
</tbody>
</table>

(inductive reasoning) = (induction)

= (statistical inference)

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**The Inferential Summary**

<table>
<thead>
<tr>
<th>Crabs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pop. Unknown pop.</td>
</tr>
<tr>
<td>Sample estimate of $M$</td>
</tr>
<tr>
<td>Imagin. data set Give-or-take for $\bar{Y}$ as est. of $M$</td>
</tr>
</tbody>
</table>

Interval 95%, Confidence

$\bar{Y} \pm 1.96 \times (0.27°C) = (24.44°C, 25.56°C)$

= $(25.0°C - 0.50°C, 25.0°C + 0.50°C)$
Population

Sample

Imaginary data set

All possible values of \( \bar{y} \)

\[
\begin{bmatrix}
25.00c \\
25.20c
\end{bmatrix}
\]

\( n \to \infty \)

long run mean (EV of \( \bar{y} \))

\( = \mu \)

long run SD (SE of \( \bar{y} \))

\( = \frac{s}{\sqrt{n}} = \frac{1.34^0c}{\sqrt{25}} = 0.27^0c \)

long run histogram of \( \bar{y} \)

\( \bar{SE} = 0.27^0c \)

Hypothetical samples

\( \bar{y} \) of \( \bar{y} \) as an estimate of \( \mu \)

Expected value = EV of \( \bar{y} \) = \( \mu \)

Standard error of \( \bar{y} \) = SE of \( \bar{y} \) = SE IID (\( \bar{y} \)) = \( \frac{s}{\sqrt{n}} \)

\( L \to \bar{SE} (\bar{y}) = \frac{s}{\sqrt{n}} = \frac{1.34}{\sqrt{25}} = 0.27^0c \)

\( \bar{y} \bar{y} \) this is the estimated SE of \( \bar{y} \)

State of statistical inference as of 1907

* William Gosset (1908)

L worked for Guinness Brewery

* Hops, yeast, barley, water = Beer!

* He noticed a flaw with the model

long run histogram of \( \bar{y} \) (t curve)

accounting for uncertainty in \( \sigma \)

\( \bar{SE} 0.27^0c \)

\( t_{n-1} \)

degrees of freedom
\[ \text{Table} \]

- \( n = 25 \)
- \( n - 1 = 24 \)

\[ \text{normal curve} \]

\[ \text{as } Y \to \infty \]

\[ \text{t}_{24} \text{ curve} \]

\[ -2.064 \quad 2.064 \]

\[ \text{long run histogram of } \bar{Y}, \text{ accounting for uncertainty in } \sigma \]

\[ SE = 0.27^\circ C \]

\[ M \pm 2.064 \hat{SE} = M \pm (2.064)(0.27^\circ C) \]

\[ P_F \left( M - 2.064 \hat{SE} \leq \bar{Y} \leq M + 2.064 \hat{SE} \right) = 95\% \]

\[ \text{relative frequency} \]

\[ \text{(probability)} \]

\[ \text{Neymann's confidence trick} \]

\[ P_F \left( \bar{Y} - 2.064 \hat{SE} \leq M \leq \bar{Y} + 2.064 \hat{SE} \right) = 95\% \]

\[ \text{lets use } \bar{Y} \pm \frac{2.064 \hat{SE}}{\text{t}_{n-1} 0.95} \text{ as a 95\% confidence interval (for } M) \]

\[ 24.3^\circ C \]

\[ \begin{array}{c}
M_0 \quad 29.94^\circ C \\
29.0^\circ C \quad 25.0^\circ C \\
25.5^\circ C
\end{array} \]

\[ \rightarrow 95\% \text{ CI for } M \]

\[ * \text{at 95\% level of confidence, the theory value of } 24.3^\circ C \text{ is not supported by the data; The diff. between } \bar{Y}(250^\circ C) - 8 M_0 (24.3^\circ C) \ldots \]
is statistically significant (statsig).

\( M_0 = \) theory value of \( M = 24.3^\circ C \)