this time: prob. models for sums, means; next time: inference

read: DD ch. 1–11 (B), 1–3 (A)

LN pp. 127 → 156

AMS 7 1 May 18

today ①

TH midterm handed out on 3 May in class (L-119)

due Sun 13 May at 11.59 pm

I will hold extra office hours Fri, Sat, Sun, Mon, ... in Jack's Lounge

Roulette

people try to maximize average utility = (money, satisfaction) function

(R-52)

$$\sigma = \sqrt{\frac{\left[(-\$1) - (-\$0.05)\right]^2 \cdot 37 + 1\left[(+\$35) - (-0.05)\right]^2}{38}}$$

math fact: if pop contains only 2 values,

$$\sigma = \left(\begin{array}{c}\text{larger}\\ \text{value}\end{array}\right) - \left(\begin{array}{c}\text{smaller}\\ \text{value}\end{array}\right) \sqrt{\left(\begin{array}{c}\text{proportion}\\ \text{of}\\ \text{larger}\\ \text{values}\end{array}\right)\left(\begin{array}{c}\text{proportion}\\ \text{of}\\ \text{smaller}\\ \text{values}\end{array}\right)}$$

here, $\sigma = \underbrace{[(\$+35) - (\$-1)]}_{\$36} \sqrt{\frac{1}{38} \cdot \frac{37}{38}}$

$= \$5.76$

on average,

on each \$1 bet on a single #,

I expect to (win about $\mu = \$-0.05$)
(lose \$0.05) —)

give or take about $\sigma = \$5.76$

(your net gain after 1,000 \$1 bets on a single #)	is like	(the sum S of $n = 1{,}000$ IID draws from pop.^{n})
the world		our probability model of the world

P(coming out ahead) = P($S > \$0$) = ?

p* ~~pop~~ possible spins

(A): single #: $\boxed{6}$

~~sample~~ ~~the~~ observed spins

~~imaginary~~ data set (3)

~~all possible~~ values of S

your net gain

N = 38

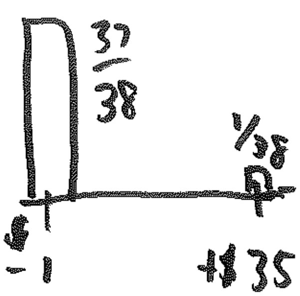
37 - \$1,

1 \$+35

IID

mean $\mu = \$-0.0526$

SD $\sigma = \$5.76$



your net gain

-1 +35

mostly -\$1, every so often \$+35

n = 1,000

sum S = ? ex. \$-64

IID

n = 1,000 \$1,000

sum S = ? ex. \$-28

\$-64 a S

-28 another S

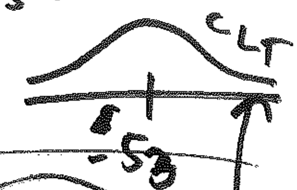
:

n → ∞

long run mean — expected value of S = -\$53

long run SD — standard error of S = \$182

long run hist — SE \$182



$\frac{1}{38} \cdot 1000 \doteq 26$ wins → \$910

974 losses -974

$\overline{1000}$ -64

27 wins → +945

973 losses -973

-28

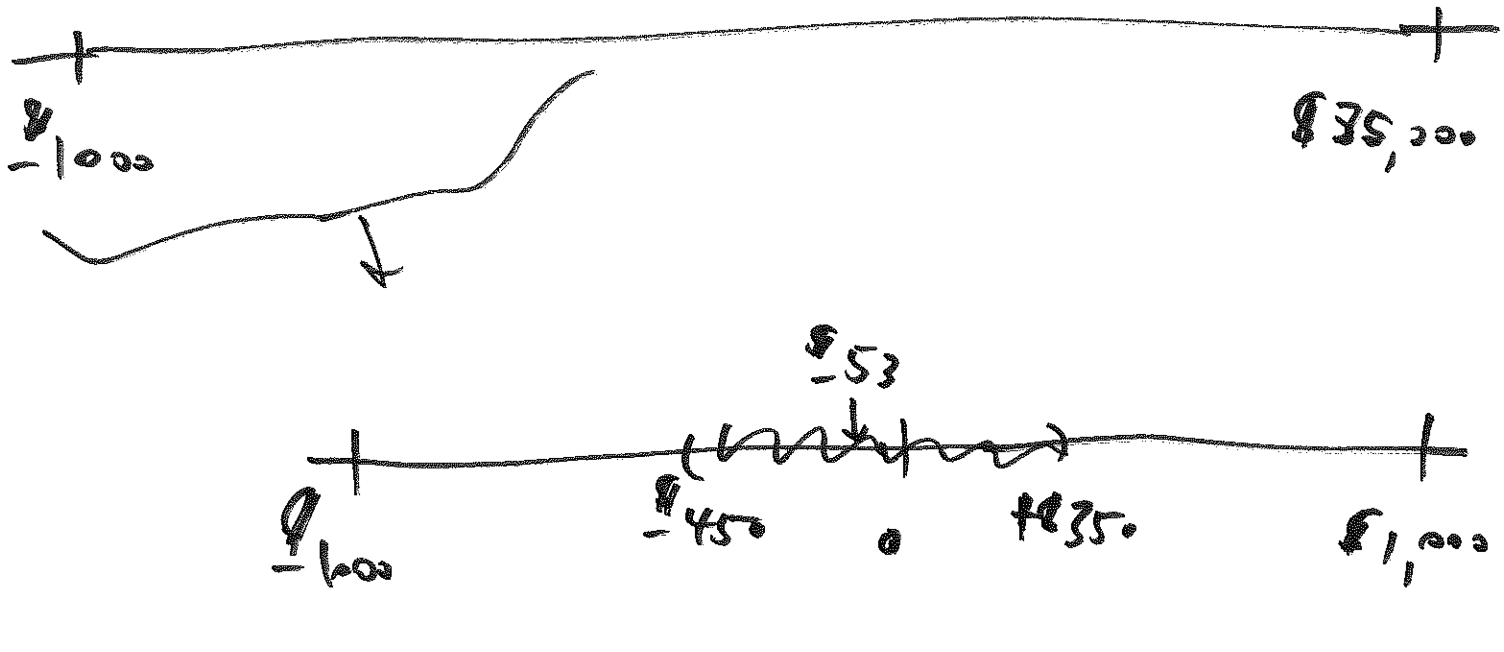
$$\begin{pmatrix}\text{expected}\\ \text{value}\\ \text{of } S\end{pmatrix} = \text{EV of } S = \boxed{E_{IID}(S) = n\mu}$$

$$= \begin{pmatrix}\#\\ \text{draws}\end{pmatrix} \cdot \begin{pmatrix}\text{pop}\\ \text{mean}\end{pmatrix}$$

possible ingredients	belong?
N	no
μ	$\mu \uparrow$ $S \uparrow$
σ	X
n	✓ yes

here

$$E_{IID}(S) = (1000)(-\$0.0526) = -\$52.60$$



after $n = 1{,}000$ \$1 plays on a single #, (5)

you expect to have won about (~~lost about \$53~~)

$E_{IID}(S) = -\$53$, give or take

about $SE_{IID}(S) = \$182$

$\begin{pmatrix}\text{standard} \\ \text{error} \\ \text{of } S\end{pmatrix} = SE \text{ of } S = SE_{IID}(S)$

$SE_{IID}(S) = \sigma\sqrt{n}$

(\$) (\$)

σ = "noise level" in each draw from pop.

possible ingredients	belong!
N	no
μ	no
σ	$\sigma\uparrow SE\uparrow$
n	$n\uparrow SE\uparrow$

$SE(S) = \begin{pmatrix}\text{pop} \\ SD\end{pmatrix}\sqrt{\#\text{ draws}} = (\$5.76)\sqrt{1000} \doteq \$182$

⑥

SE \$182

-\$53 \$0

EV

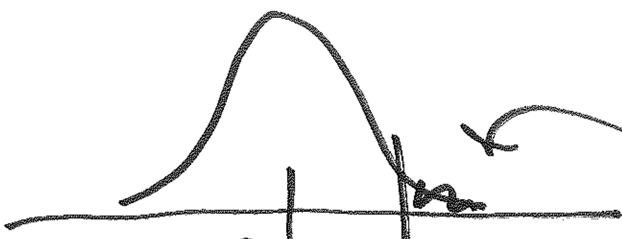
SU

long run hist of \$ $\boxed{A}$

29% = P(coming out ahead, strategy $\boxed{A}$)

$$\frac{\$0-(-\$53)}{\$182} = +0.29$$

SE \$127



-\$53 \$0

0.42

ditto $\boxed{B}$

P(\$ > 0) ≑ 34%