this time: probability

homework 2 due by 11:59pm on Fri 4 May 18

AMS 7 24 Apr 18

①

next time: inference

Thomas Bayes (~1700 → ~1760)

Pascal / Fermat (1660)

population $\xrightarrow[\text{random}]{\text{at}}$ sample

$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} Y_1 \end{bmatrix}$ $n = 1$

$P(Y_1 \text{ is odd}) = ?$

ELM? yes, because sampling was at random

$P(Y_1 \text{ is odd}) = \frac{2}{3}$

PF

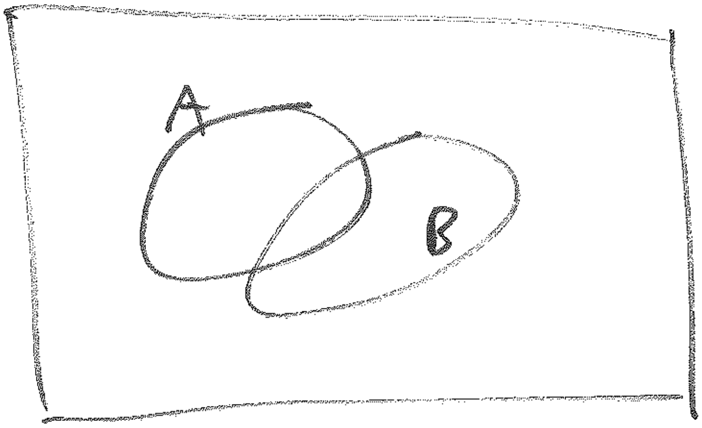
in genetics study. ELM? yes, so

$P(\text{any one of their children is T-S}) = \frac{1}{4} = 25\%$

$P(A)$, $P(\text{not } A)$?

$P(A \text{ or } B) = P(A) \; ? \; P(B)$

$P(A \text{ and } B)$



$\boxed{A}$

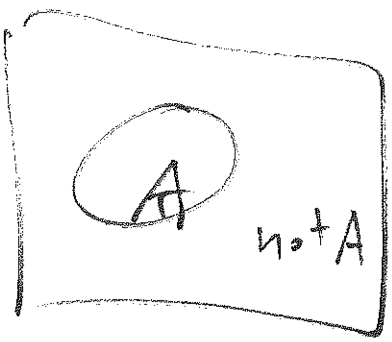
$$P(A) = \frac{A}{\boxed{1}}$$

Easy rule:

1 for any T/F statement A,

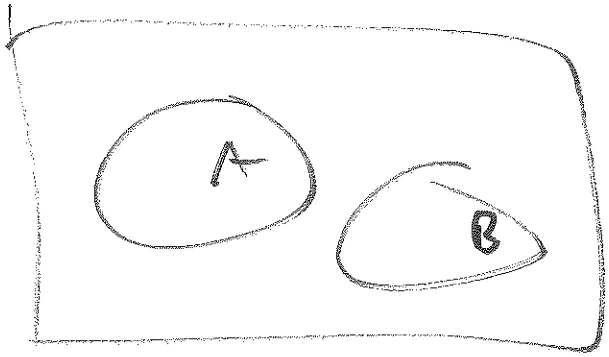
$$0\% = 0 \le P(A) \le 1 = 100\%$$

2



$$P(A) + P(\text{not } A) = 1 = 100\%$$

$$P(A) = 1 - P(\text{not } A)$$

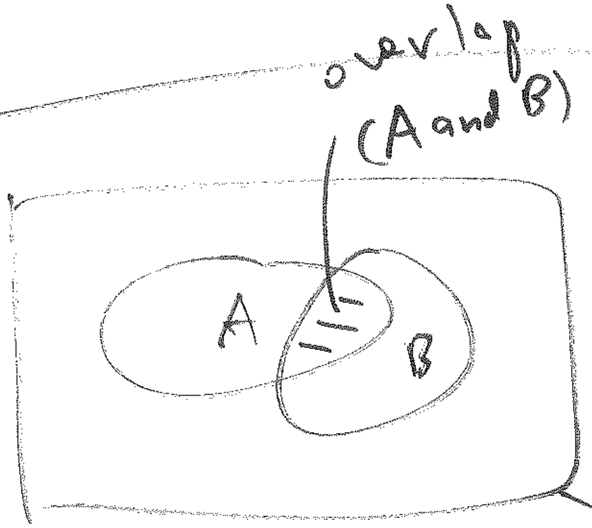


(for this diagram)

$P(A \text{ or } B) =$

$P(A) + P(B)$

addition rule for or with mutually exclusive A, B



for this diagram

$P(A \text{ or } B) =$

$P(A) + P(B)$

$- P(A \text{ and } B)$

general addition rule for OR

and

pop.

sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad n = 2$$

$P(y_1 = 2 \ \underline{\text{and}} \ y_2 = 2) = \ ?$

pop

sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \quad \text{at random with replacement} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad n = 2$$

IID sampling

independent
identically
distributed

$P(X_1 = 2 \text{ and } X_2 = 2) = ?$

IID

ELM? yes

		X_2: 1	X_2: 2	X_2: 9
X_1	1	(1, 1)	(1, 2)	(1, 9)
	2	(2, 1)	(2, 2)	(2, 9)
	9	(9, 1)	(9, 2)	(9, 9)

$P(X_1 = 2 \text{ and } X_2 = 2) =$

$$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

$\begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix}$ $1/3$

$P(X_1 = 2) = \frac{3}{9} = \frac{1}{3}$

$P(X_2 = 2) = \frac{3}{9} = \frac{1}{3}$

conjecture:

$P(A \text{ and } B) = P(A) \cdot P(B)$

P(1 or more T-5)

T-5 babies

~~if ELM~~

0

not equally likely: 1, 2, 3, 4, 5

$P(1 \text{ or more}) = \frac{5}{6}$

pop

sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

at random without replacement

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad n = 2$$

simple random sampling (SRS)

common sense

$$P(X_1 = 2 \text{ and } X_2 = 2) = 0$$

$$P(X_1 = 2) = \frac{1}{3} \; ; \; P(X_2 = 2) = \frac{1}{3}$$

SRS

		X_2: 1	2	9
X_1	1	~~(1,1)~~	(1,2)	(1,9)
	2	(2,1)	~~(2,2)~~	(2,9)
	9	(9,1)	(9,2)	~~(9,9)~~

ELM? yes

$$P(X_2 = 2) = \frac{2}{6} = \frac{1}{3}$$

(R)

here $P(X_1 = 2 \text{ and } X_2 = 2) = 0$

but $P(X_1 = 2) \cdot P(X_2 = 2) = \frac{1}{9} \neq 0$

$$\frac{1}{3} \cdot \frac{1}{3}$$

we need a new kind of probability for <u>dependence</u>: conditional probability