This probability time:

next inference time:

Thomas Bayes
(-1701 - 1761)

Pascal / Fermat (1660)

Population sample

\[
\begin{pmatrix}
1 \\
2 \\
9
\end{pmatrix}
\xrightarrow{\text{random}} \left[ X_i \right]_{i=1}^n
\]

\[P(\text{I}_i \text{ is odd}) = \frac{1}{3}\]

\[P(\text{I}_i \text{ is even}) = \frac{2}{3}\]

\[
P(\text{E} | \text{M} ?) \text{ yes, because sampling was at random}
\]

in general

\[P(A, \text{ not } A) = P(A) + P(\text{not } A)\]

\[P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)\]

\[P(\text{any one of their children is T-S}) = \frac{1}{4} = 25\%\]
Easy rule:

1. For any True/False statement $A$,
   
   $0 \% = 0 \leq P(A) \leq 1 = 100 \%$

2. $P(A) + P(\neg A) = 1$
   
   $P(A) = 1 - P(\neg A)$
for this diagram

\[ P(A \text{ or } B) = P(A) + P(B) \]

addition rule for \( \bigcirc \) with mutually exclusive \( A, B \)

for this diagram

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

several addition rule for \( \bigcirc \)

\[ \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \]

\[ P(\xi_1 = 2 \text{ and } \xi_2 = 2) = ? \]
I.I.D. Sampling

\[ \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 2 \\ 9 \end{pmatrix} \]

\[ \text{random with replacement} \]

\[ \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \]

\[ P(\overline{B_1} = 2 \text{ and } \overline{B_2} = 2) = ? \]

\[ P(\overline{B_1} = 2) = \frac{3}{9} = \frac{1}{3} \]

\[ P(\overline{B_2} = 2) = \frac{3}{9} = \frac{1}{3} \]

Conjecture:

\[ P(A \cup B) = P(A) \cdot P(B) \]
\[ P(\mathbb{E}_1 = 2 \text{ or } \mathbb{E}_2 = 2) = 0 \]

\[ P(\mathbb{E}_1 = 2) = \frac{1}{3} \quad P(\mathbb{E}_2 = 2) = \frac{1}{3} \]

\[ \begin{array}{ccc}
1 & 1 & 2 \\
1 & 9 & 1
\end{array} \]

Ecm: Yes

\[ P(\mathbb{E}_2 = 2) = \frac{2}{6} = \frac{1}{3} \]

Here \[ P(\mathbb{E}_1 = 2 \text{ and } \mathbb{E}_2 = 2) = 0 \]

But \[ P(\mathbb{E}_1 = 2) \cdot P(\mathbb{E}_2 = 2) = \frac{1}{9} \neq 0 \]

\[ \frac{1}{3} \quad \frac{1}{3} \]
we need a new kind of probability for dependence: Conditional probability