Conditional probability

Next prob. under: for sums

Conditional probability

\[ P(A) = \frac{A}{1} \]

\[ P(A \text{ given } B) = ? \]

\[ P(A \mid B) = ? \]

**def:** (de Moivre 1705)

\[ P(A \mid B) = \begin{cases} \frac{P(A \text{ and } B)}{P(B)} & \text{if } P(B) > 0 \\ \text{undefined} & \end{cases} \]
\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

\[ P(B) \cdot P(A \mid B) = P(A \cap B) \]

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\[ P(Z_1 = 2 \text{ and } Z_2 = 2) = \frac{5}{5} \]

\[ P(Z_2 = 2) \cdot P(Z_2 = 2 \mid Z_1 = 2) = \frac{1}{3} \cdot 0 = 0 \]
\[ P(Z_1 = 2 \text{ or } Z_2 = 2) = \]
\[ = \frac{1}{2} \]
\[ = P(Z_1 = 2) \cdot P(Z_2 = 2) \]

**def:** \( A, B \) are independent if & only if information about \( A \) doesn't change chances of \( B \) & vice versa:

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]
P(1 or more T-s alleles in family of 5, both parents carriers) = 1 - P(0 T-s alleles)

= 1 - P(\frac{15}{\text{not T-s}} \cdot \frac{\text{not and}}{2 \text{ and}} \cdot \frac{5+2}{\text{not T-s}})

= 1 - P\left(\frac{\text{not T-s}}{\frac{15}{20}} \cdot \frac{\text{not T-s}}{\frac{20-5}{20}} \cdot \frac{5+2}{\text{not T-s}}\right)

\Box

= 1 - (1 - \frac{1}{4}) \cdot (1 - \frac{1}{4}) \cdot (1 - \frac{1}{4})

= 1 - (1 - \frac{1}{4})^5 = 0.76 = 76\%

UCLA marijuana case study

\(\Box 37\)
\[
P(Y) = \frac{81}{106} = 76\%
\]
\[
P(Y | F) = \frac{29}{49} = 59\%
\]
\[
P(Y | M) = \frac{52}{57} = 91\%
\]
Are gender & NLP independent or dependent in this dataset?

[A: dependent (G & NLP are associated)]

91%

76% → 91%

91% differs from 59% by a

amount that has in practical
terms (i.e., practically significant)

Pearson's study: \( r = 51 \)
\[ P(DP | DW) = \frac{19}{160} = 11.9\% \]

\[ P(DP | DB) = \frac{12}{166} \]

\[ P(DP) = \frac{36}{326} = 11\% \]

\[ P(DP | VW) = \frac{30}{214} = 14.0\% \]

\[ P(DP | DW \circ VW) = \frac{19}{157} = 12.6\% \]

\[ P(DP | DB \uparrow VW) = \frac{11}{63} = 17.5\% \]
bottom\table\[ P(\text{VP} \mid \text{VB}) = \frac{6}{112} = 5.4\% \]

victim block

\[ P(\text{DP} \mid \text{VB, DW}) = \frac{0}{9} = 0\% \]

\[ P(\text{DP} \mid \text{VB, DB}) = \frac{6}{103} = 5.8\% \]

direction of relationship between $x$ & $y$ changes when $z$
is accounted for: Simpson's $\text{(1950)}$

R_{xyz}^2

Probabilty model for sums & means

Roulette ($P(\frac{52}{1})$)
On average, I expect to lose 1.8 points at 8.1 per match. In a single play, I expect to lose 1.8 points at 8.1 per match. On average, I expect to lose 8.1 points at 1.8 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match. For each 8.1, I expect to lose 1.8 points at 8.1 per match.