AMS 7
3 May 18

this time: prob. models for [redacted] means

next time: inference

get more good data ← unbiased (1)

measurement error

$$\begin{bmatrix}16\\16\\16\\\vdots\\16\end{bmatrix}\text{ oz.}\quad\begin{bmatrix}16.0\\16.0\\16.0\\\vdots\\16.0\end{bmatrix}\qquad\begin{bmatrix}16.03\\15.99\\15.95\\\vdots\end{bmatrix}\ \ldots$$

non-random (deterministic)

probabilistic (random) (stochastic)

astronomy (1600s)

basic measurement error model

$$\begin{pmatrix}\text{obs.}\\\#1\end{pmatrix}=\begin{pmatrix}\text{true}\\\text{value}\end{pmatrix}+(\text{bias})+\begin{pmatrix}\text{random}\\\text{error}\\\#1\end{pmatrix}$$

$$\begin{pmatrix}\text{obs}\\\#2\end{pmatrix}=\begin{pmatrix}\text{true}\\\text{value}\end{pmatrix}+(\text{bias})+\begin{pmatrix}\text{random}\\\text{error}\\\#2\end{pmatrix}$$

$$\vdots\qquad\qquad\vdots\qquad\qquad\vdots\qquad\qquad\vdots$$

IID draws from a normal

$$\begin{pmatrix} \text{obs} \\ \# n \end{pmatrix} = \begin{pmatrix} \text{true} \\ \text{value} \end{pmatrix} + (\text{bias}) + \begin{pmatrix} \text{random} \\ \text{error} \\ \# n \end{pmatrix}$$ ②

theta

Curve with mean 0 & SD σ

$$Y_1 = \theta + b + e_1$$

$$Y_2 = \theta + b + e_2$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$Y_n = \theta + b + e_n$$

(oz) (unbiased)

$$16.03 = 16.0 + 0 + (+.03)$$

$$(Y_1) = \theta + b + e_1$$

$$15.99 = 16.0 + 0 + (-0.01)$$

$$(Y_2) \quad \theta + b + e_2$$

$\vdots$

$$15.95 = \theta + b + (-0.05)$$

$$(Y_n) \quad 16.0 + 0 + e_n$$

③

$$Y_1 = \theta + b + e_1$$ ← IID normal with mean 0

$$Y_2 = \theta + b + e_2$$

$$\vdots$$

$$Y_n = \theta + b + e_n$$

average (mean)

$$\bar{Y} = \theta + b + \bar{e}$$

$\bar{e}$ = (mean of n IID draws, each with mean 0)

$\bar{e}$ will (with high prob.) be closer to 0 than any of the e_i

ex.

$$\frac{(+0.03) + (-0.01) + \ldots + (-0.05)}{n}$$

as $n \uparrow$, $\bar{e} \downarrow 0$ with high prob.; (4)

therefore, $\bar{y}$ will be close (when n is large) to $(\theta + b)$
(truth + bias)

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to make $\bar{y}$ get arbitrarily close to θ, we need 2 things:

(1) n should get $\uparrow$ &

(2) $b = 0$ (measuring process is <u>unbiased</u>)

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you cannot make bias $\downarrow 0$ as n increases.

1936 Literary Digest (LD)

Roosevelt (D)
Landon (R)

sent out 24,000 letters (pre-stamped post card)

got back 16,000 post cards:

LD est. that
Landon 60%
Roosevelt 40%

truth:
Roosevelt 60%
Landon 40%

20-percentage point error

George Gallup (Iowa state) (1,000 people)

addresses? 1936
- landowner records (rich)
- telephone books (rich)
- (v. rich) club membership lists

(6)

pop (conceptual) all possible measurements

sample the observed measurements

imaginary data set all possible $\bar{y}$ values

potassium level

$N = \infty$

actual ~~like~~ IID

potassium level

y_1, y_2, $\vdots$, y_4

$n = 4$

mean $\bar{y} = ?$ (ex. 3.92)

3.92 a $\bar{y}$

3.77 actual $\bar{y}$

$\vdots$

$M \to \infty$

hyp. IID

SD $s =$

$n = 4$

mean $\bar{y} = ?$ (ex. 3.77)

$\vdots$

mean $\mu = 3.8$

SD $\sigma = 0.2$

SD 0.2

pop. hist.

3.8

long run mean | expected value of $\bar{y}$

long run SD | standard error SD of $\bar{y}$

long run hist | SE 0.1 CLT

3.8

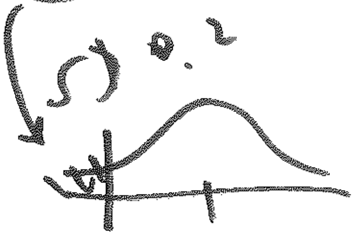
EV

$$\left(\begin{array}{c}\text{expected}\\ \text{value}\\ \text{of } \bar{y}\end{array}\right) = \left(\text{EV of } \bar{y}\right) = \boxed{E_{IID}(\bar{y}) = \mu}$$

math fact $= 3.8$

7%

σ) 0.2



pop. hist. = hist of one reading at a time (Y_1)

$\boxed{\text{L-133}}$

3.5 3.8

$$\frac{3.5 - 3.8}{0.2} = \frac{-0.3}{0.2} = -1.5$$

P(misdiagnosis with $n = 1$) $\doteq$ 7%

error rate too high

P(misdiagnosis with $n = 4$) = P($\bar{Y}_{(n=4)} < 3.5$) = ?

$$\left(\text{standard error of } \bar{Y} \right) = \left(SE \text{ of } \bar{Y} \right) \to SE_{IID}(\bar{Y}) = ?$$

$$\boxed{SE_{IID}(\bar{Y}) = \frac{\sigma}{\sqrt{n}}}$$

square root law

uncertainty

goes down with n, but only at a $\sqrt{n}$ rate

ingredient	below?
N	X
μ	X
σ	$\sigma \uparrow$ $SE(\bar{Y}) \uparrow$
n	$n \uparrow$ $SE(\bar{Y}) \downarrow$

⑧

square root law

to cut $SE(\bar{y})$ in half, need to quadruple n

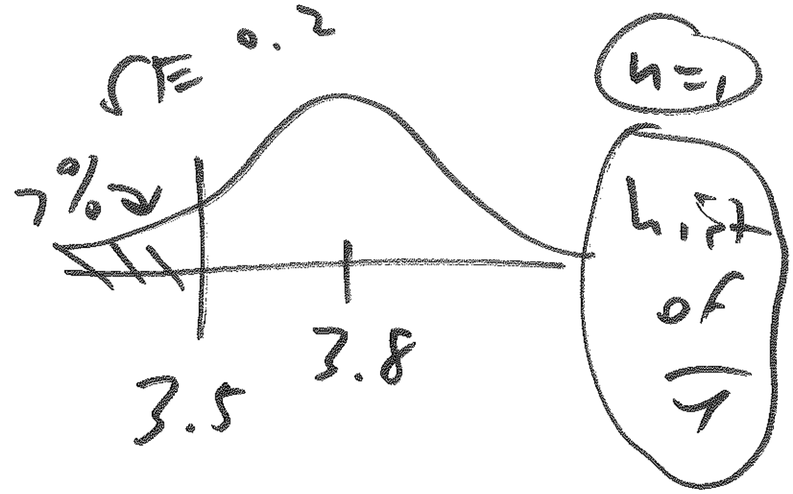
$$SE(\bar{y}) = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{4}} = 0.1$$

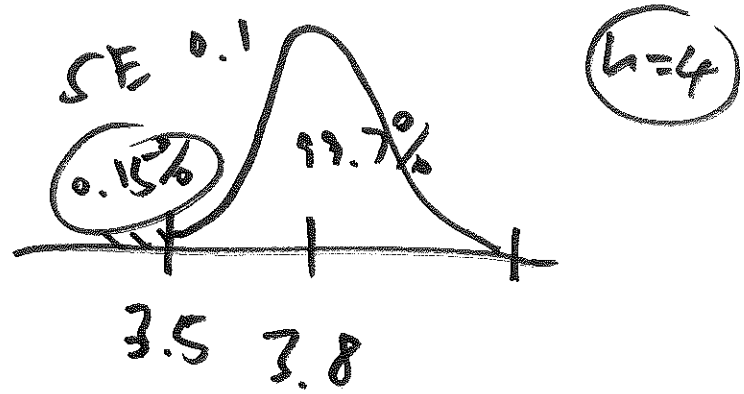
here

L-124 | CLT

n	P(mis diag.)	Cost
1	7%	\$25
4	0.15%	\$100

cost-benefit tradeoff





$$\frac{3.5 - 3.8}{0.1} = \frac{-0.3}{0.1} = -3$$